Maths Readiness Check

B3MIN1051 - Financial Decision Making

Getting started

Purpose

This maths readiness check does not require any finance knowledge. It focuses on the calculations and algebraic manipulation used throughout the course *B3MIN1051 – Financial Decision Making.*

How to use this check

- Try to complete the questions without notes first.
- If you get stuck, use your old high-school maths notes or consolidate online resources.
- You may use a calculator for arithmetic, logarithms, and powers. A non-programmable calculator is allowed in the final exam.
- Worked solutions are provided at the end. Check them only after attempting each question.
- If you can solve the questions, you are well prepared for the mathematical skills required in the course. If several topics feel difficult (e.g., logs/exponents or basic statistics), review those areas before the course begins.

Conventions

Unless stated otherwise, round final numerical answers to **4 decimal places**. $\ln(\cdot)$ refers to the natural logarithm.

A few helpful formulas

- Percentage change: $\%\Delta = \dfrac{\text{new} - \text{old}}{\text{old}} \times 100\%$.
- Quadratic formula for $ax^2 + bx + c = 0$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- Sample mean: $\bar{x} = \dfrac{1}{n} \sum_{i=1}^{n} x_i$.
- Sample variance: $s_x^2 = \dfrac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$.
- Sample covariance: $s_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$.
- Correlation: $\rho_{xy} = \dfrac{s_{xy}}{s_x s_y}$.

Questions

A. Arithmetic, percentages, and ratios

1. Convert (i) 7.5% to a decimal and (ii) 0.042 to a percentage.
2. Compute 12% of 250.
3. A value changes from 80 to 92. Compute the percentage change.
4. Solve for x: $\quad 1.08x = 540$.
5. Compute the weighted average of 3 and 7 with weights 40% and 60%.
6. Suppose $\frac{x}{y} = 0.75$ and $x + y = 140$. Find x and y.
7. A quantity increases by 5% and then decreases by 5%. What is the net percentage change?

B. Algebra and rearranging equations

8. Solve for x: $\quad 3(2x - 5) = 4x + 7$.
9. Rearrange $y = a + bx$ to solve for b (assume $x \neq 0$).
10. Simplify: $\quad \dfrac{x^{-2}y^3}{x^3y^{-1}}$.
11. Solve the system:

$$\begin{cases} 2x + 3y = 12, \\ -x + 2y = 3. \end{cases}$$

12. A weighted mean is defined as $\bar{z} = \dfrac{w_1z_1 + w_2z_2}{w_1 + w_2}$. Given $z_1 = 4$, $w_1 = 2$, $w_2 = 3$, and $\bar{z} = 7$, solve for z_2.

C. Powers, roots, and logarithms

13. Compute $(1.03)^5$.
14. Solve for T: $\quad 500(1.04)^T = 800$.
15. Solve for r: $\quad (1 + r)^{10} = 1.5$.
16. Effective compounding: compute

$$(1 + 0.06/12)^{12} - 1.$$

17. A quantity changes from 50 to 60.5 over 2 equal time periods. Compute the constant per-period growth rate g such that

$$50(1 + g)^2 = 60.5.$$

18. Suppose $F(r,T) = \dfrac{1-(1+r)^{-T}}{r}$. Compute $F(0.05, 10)$, i.e. F with $r = 0.05$ and $T = 10$.

19. Suppose that $S = C \cdot \dfrac{1-(1+r)^{-T}}{r}$ and $S = 5000$, $r = 0.05$ and $T = 10$. Find C.

20. Suppose $S = C \cdot \dfrac{1-(1+r)^{-T}}{r}$ with $S = 3000$, $C = 450$, $r = 0.04$. Solve for T.

D. Quadratics and solving nonlinear equations

21. Solve $2x^2 - 5x - 3 = 0$.

22. Solve for x (assume $x > -1$):

$$90 = \frac{40}{1+x} + \frac{60}{(1+x)^2}.$$

(*Hint:* let $y = \frac{1}{1+x}$.)

E. Data, probability, and statistics

23. For the data $[2, 4, 6, 8]$, compute the sample mean, sample variance, and sample standard deviation.

24. A discrete random variable X takes values $0, 1, 2$ with probabilities $0.2, 0.5, 0.3$ respectively. Compute the sample mean ($\mathbb{E}[X]$)

Solutions

A. Arithmetic, percentages, and ratios

1. *Converting between percentages and decimals.* A percentage means "per 100".

$$7.5\% = \frac{7.5}{100} = 0.075.$$

To convert 0.042 to a percentage, multiply by 100:

$$0.042 \times 100\% = 4.2\%.$$

2. *Finding a percentage of a number.*

$$12\% \text{ of } 250 = 0.12 \times 250 = 30.$$

3. *Percentage change.* Use $\%\Delta = \dfrac{\text{new} - \text{old}}{\text{old}} \times 100\%$:

$$\%\Delta = \frac{92 - 80}{80} \times 100\% = \frac{12}{80} \times 100\% = 0.15 \times 100\% = 15\%.$$

4. *Solving a one-step linear equation.* Divide both sides by 1.08:

$$x = \frac{540}{1.08} = 500.$$

5. *Weighted average.* A weighted average is $\sum w_i x_i$ where weights sum to 1.

$$0.4 \cdot 3 + 0.6 \cdot 7 = 1.2 + 4.2 = 5.4.$$

6. *Ratios and totals.* From $\frac{x}{y} = 0.75$ we get $x = 0.75y$. Substitute into $x + y = 140$:

$$0.75y + y = 1.75y = 140 \quad \Rightarrow \quad y = \frac{140}{1.75} = 80.$$

Then $x = 0.75 \cdot 80 = 60$.

7. *Sequential percentage changes.* An increase by 5% multiplies by 1.05, then a decrease by 5% multiplies by 0.95:

$$1.05 \times 0.95 = 0.9975.$$

So the final level is 0.9975 of the original, i.e. a net change of

$$(0.9975 - 1) \times 100\% = -0.25\%.$$

B. Algebra and rearranging equations

8. *Solving a linear equation with brackets.* Expand the left-hand side:

$$3(2x-5)=6x-15.$$

Set equal to the right-hand side and collect terms:

$$6x-15=4x+7 \;\Rightarrow\; 2x=22 \;\Rightarrow\; x=11.$$

9. *Rearranging to isolate a variable.* Starting from $y=a+bx$:

$$y-a=bx \quad \Rightarrow \quad b=\frac{y-a}{x} \quad (x\neq 0).$$

10. *Simplifying powers.* Use $x^m/x^n=x^{m-n}$ and $y^m/y^n=y^{m-n}$:

$$\frac{x^{-2}y^3}{x^3y^{-1}}=x^{-2-3}\,y^{3-(-1)}=x^{-5}y^4=\frac{y^4}{x^5}.$$

11. *Solving a 2×2 system.* From the second equation:

$$-x+2y=3\Rightarrow x=2y-3.$$

Substitute into the first:

$$2(2y-3)+3y=12\Rightarrow 4y-6+3y=12\Rightarrow 7y=18\Rightarrow y=\frac{18}{7}.$$

Then

$$x=2\left(\frac{18}{7}\right)-3=\frac{36}{7}-\frac{21}{7}=\frac{15}{7}.$$

Numerically: $y\approx 2.5714$, $x\approx 2.1429$.

12. *Solving for an unknown in a weighted mean.* Given

$$\bar{z}=\frac{w_1z_1+w_2z_2}{w_1+w_2},$$

plug in values:

$$7=\frac{2\cdot 4+3z_2}{2+3}=\frac{8+3z_2}{5}.$$

Multiply by 5 and solve:

$$35=8+3z_2\Rightarrow 3z_2=27\Rightarrow z_2=9.$$

C. Powers, roots, and logarithms

13. *Computing a power.*

$$(1.03)^5\approx 1.1593.$$

(You can compute this directly on a calculator.)

14. *Solving an exponential equation for the exponent.*

$$500(1.04)^T = 800 \Rightarrow (1.04)^T = \frac{800}{500} = 1.6.$$

Take logs:

$$T \ln(1.04) = \ln(1.6) \Rightarrow T = \frac{\ln(1.6)}{\ln(1.04)} \approx 11.9836.$$

15. *Solving an exponential equation for the base.*

$$(1+r)^{10} = 1.5 \Rightarrow 1+r = 1.5^{1/10} \Rightarrow r = 1.5^{1/10} - 1 \approx 0.0414.$$

So $r \approx 4.1379\%$.

16. *Repeated compounding.*

$$(1+0.06/12)^{12} - 1 = (1.005)^{12} - 1 \approx 1.0617 - 1 = 0.0617.$$

So the result is about 6.1678%.

17. *Constant growth rate over multiple periods.*

$$50(1+g)^2 = 60.5 \Rightarrow (1+g)^2 = \frac{60.5}{50} = 1.21.$$

Take square roots (use the positive root because $1+g>0$):

$$1+g = \sqrt{1.21} = 1.1 \Rightarrow g = 0.1.$$

So the per-period growth rate is 10%.

18. *Working with the factor* $F(r,T) = \dfrac{1-(1+r)^{-T}}{r}$. Compute $F(0.05, 10)$:

$$F(0.05, 10) = \frac{1-(1.05)^{-10}}{0.05} \approx 7.7217.$$

19. *Working with* $S = C \cdot \dfrac{1-(1+r)^{-T}}{r}$. If $S = 5000$, $r = 0.05$ and $T = 10$, then

$$C = \frac{S}{\frac{1-(1.05)^{-10}}{0.05}} = \frac{5000}{7.7217} \approx 647.5229.$$

20. *Working with* $S = C \cdot \dfrac{1-(1+r)^{-T}}{r}$. If $S = 3000$, $C = 450$ and $r = 0.04$, then

$$3000 = 450 \cdot \frac{1-(1.04)^{-T}}{0.04}$$

$$\frac{3000}{450} = \frac{1-(1.04)^{-T}}{0.04} \quad \Rightarrow \quad 6.66666\ldots = \frac{1-(1.04)^{-T}}{0.04}.$$

Multiply by 0.04:

$$0.266666\ldots = 1-(1.04)^{-T} \quad \Rightarrow \quad (1.04)^{-T} = 0.733333\ldots$$

Take logs:

$$-T\ln(1.04) = \ln(0.733333\ldots) \quad \Rightarrow \quad T = \frac{-\ln(0.733333\ldots)}{\ln(1.04)} \approx 7.9079.$$

D. Quadratics and solving nonlinear equations

21. *Quadratic equation.* For $2x^2 - 5x - 3 = 0$, use the quadratic formula with $a = 2$, $b = -5$, $c = -3$:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5 \pm 7}{4}.$$

So $x = 3$ or $x = -0.5$.

22. *Nonlinear equation via substitution.* Let $y = \dfrac{1}{1+x}$ (note $x > -1$ ensures $1 + x > 0$ and $y > 0$). Then

$$90 = 40y + 60y^2 \;\Rightarrow\; 60y^2 + 40y - 90 = 0.$$

Divide by 10:

$$6y^2 + 4y - 9 = 0.$$

Quadratic formula gives

$$y = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 6 \cdot (-9)}}{2 \cdot 6} = \frac{-4 \pm \sqrt{16 + 216}}{12} = \frac{-4 \pm \sqrt{232}}{12}.$$

Since $y > 0$, take the "+" root:

$$y = \frac{-4 + \sqrt{232}}{12} \approx 0.9360.$$

Finally, $y = \frac{1}{1+x}$ implies $1 + x = \frac{1}{y}$, so

$$x = \frac{1}{y} - 1 \approx \frac{1}{0.9360} - 1 \approx 0.0684.$$

E. Data, probability, and statistics

23. *Sample mean, variance, and standard deviation.* Data: $2, 4, 6, 8$ with $n = 4$.

$$\bar{x} = \frac{2 + 4 + 6 + 8}{4} = \frac{20}{4} = 5.$$

Deviations: $-3, -1, 1, 3$. Squared deviations: $9, 1, 1, 9$ with sum 20.

Sample variance:

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = \frac{20}{3} \approx 6.6667.$$

Sample standard deviation:

$$s = \sqrt{s^2} = \sqrt{\frac{20}{3}} \approx 2.5820.$$

24. *Expectation and variance for a discrete random variable.* Values and probabilities: $0(0.2)$, $1(0.5)$, $2(0.3)$.

$$\mathbb{E}[X] = 0 \cdot 0.2 + 1 \cdot 0.5 + 2 \cdot 0.3 = 0 + 0.5 + 0.6 = 1.1.$$